SR2b · RESPONSE-SPECTRUM-3D-BRACED-FRAME

Response-spectrum analysis of a three-dimensional L-shaped braced frame

Brace + rotational-mass + tabulated-spectrum RSA verified on an L-shaped three-dimensional braced frame

PASS

error +0.108% · tol 1.5%

§2 TECHNICAL FEATURES TESTED

3-D response-spectrum analysis (RSA) | **Braces / axial-only members** (linear Truss) | Rigid floor diaphragms (Ux, Uy, Rz) | **Rotational mass (MMI)** — offset-mass condensation | Tabulated El-Centro spectrum · 5% damping | **CQC · SRSS · ABS** · axial-force recovery

§3 PROBLEM DESCRIPTION

SR2 verified the 3-D response-spectrum pipeline on a moment frame; **SR2b** takes it onto the **brace surface** and adds **rotational mass**. Adopted as **CSI SAP2000 Verification Example 1-025** with the independent solution from **Peterson (1981)**, a **three-story, L-shaped building** is framed by **four identical two-bay X-braced frames** whose columns and diagonals carry **axial force only** — every member is emitted as a **linear `element Truss`**. All mass is concentrated at a **centre-of-mass joint** per level with X and Y translational mass *and* a **rotational mass moment of inertia (MMI)** about Z; the plan is L-shaped, so the CM is offset from the centre of rigidity and the response is **translation–torsion coupled**. The excitation is the **El-Centro-derived tabulated response spectrum** in X with 5% damping, combined by **CQC, SRSS and ABS**; only the first two modes drive the RSA. The production mass path injects translational mass only, so the MMI is reproduced **physically** by four offset translational masses on the diaphragm — the de-Zhu T-matrix mass condensation (`M_red = TᵀMT`) turns them into the exact Z rotational inertia, with no production-code change. The reference is reproduced directly at its own discretisation (84 truss members, 3 concentrated story masses).

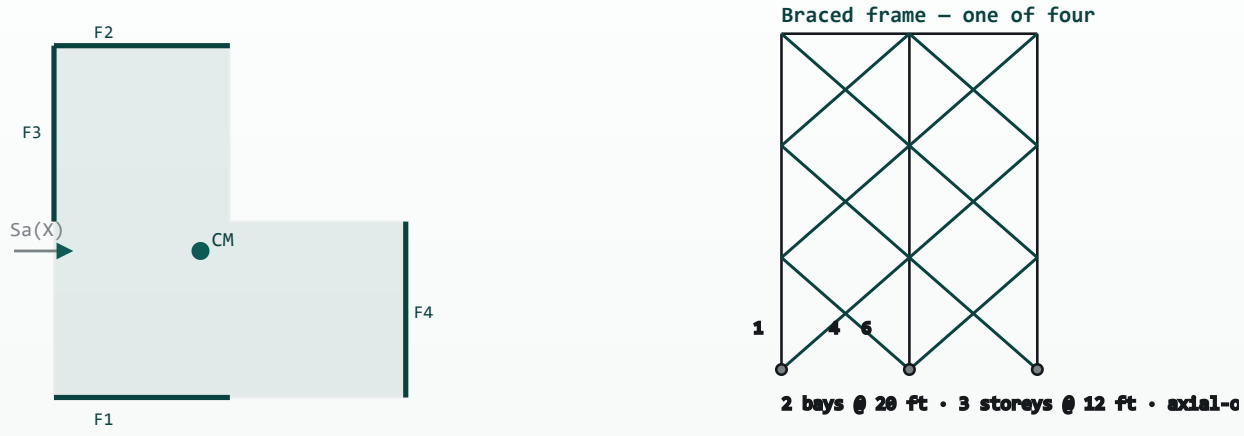

Figure SR2b. L-shaped plan with the four axial-only braced frames and the eccentric centre of mass carrying MassX, MassY and MMI (left), and the elevation of one two-bay X-braced frame with pinned, axial-only members (right). Frame-1 elements 1 (column), 4 and 6 (diagonals) are the axial-force comparison points.

§4 GEOMETRY, PROPERTIES & LOADING

GEOMETRY		MEMBERS & MASS		LOADING & DYNAMICS	
Braced frames	4	E	29500 ksi	Excitation	El-Centro spectrum (X)
Bays / frame	2	Area	6 in²	Damping	5 %
Storeys	3	Members	Axial-only (Truss)	Modes used	2
Bay / storey	20 / 12 ft	Mass	MassX,Y + MMI at CM	Combination	CQC · SRSS · ABS

§5 REFERENCE SOLUTION

Frequency — mode 1: $(K - \omega^2 M)\cdot\phi = 0$, $f = \omega/2\pi$ = 3.0592 Hz

Roof CM Ux — CQC: $u = \sqrt{\Sigma_i \Sigma_j \rho_{ij}\cdot u_i \cdot u_j}$, ρ = Der Kiureghian = 1.0329 in

Axial elm 1 — CQC: $N = \sqrt{\Sigma_i \Sigma_j \rho_{ij}\cdot N_i \cdot N_j}$ = 279.48 kip

CSI SAP2000 Software Verification, Example 1-025 (Response-Spectrum Analysis of a Three-Dimensional Braced Frame); independent solution = Peterson (1981), reproduced exactly (0 %) by SAP2000. A three-story, L-shaped building framed by four identical two-bay, three-story X-braced planar frames whose columns and diagonals carry axial force only (all pinned). The frames are tied by a rigid floor diaphragm at each level. All mass is concentrated at a centre-of-mass joint per level with X and Y translational mass (1.24224 kip-s²/in) and a rotational mass moment of inertia about Z (174,907.4 kip-in-s²), giving nine dynamic DOF; for consistency with Peterson only the first two modes drive the RSA. The excitation is the El-Centro-derived 5%-damped response spectrum applied in X, combined by CQC, SRSS and ABS. Reference values are the two modal frequencies, the roof centre-of-mass (joint 51) X/Y displacement and Z rotation, and the axial forces in three Frame-1 elements (a column and two diagonals) under each combination rule.

§6 RESULTS COMPARISON

RESPONSE QUANTITY	PROBE	STRIX	REFERENCE	DIFF	VERDICT
Frequency — mode 1 (Hz)	eigen · Hz	3.05908	3.0592	−0.0040%	PASS
Frequency — mode 2 (Hz)	eigen · Hz	3.11867	3.1188	−0.0040%	PASS
Roof CM Ux — CQC (in)	CQC · in	1.03241	1.0329	−0.048%	PASS
Roof CM Ux — SRSS (in)	SRSS · in	0.736835	0.7372	−0.05%	PASS
Roof CM Ux — ABS (in)	ABS · in	1.04181	1.0423	−0.047%	PASS
Roof CM Uy — CQC (in)	CQC · in	0.141363	0.1414	−0.026%	PASS
Roof CM Uy — SRSS (in)	SRSS · in	0.736835	0.7372	−0.05%	PASS
Roof CM Uy — ABS (in)	ABS · in	1.04181	1.0423	−0.047%	PASS
Roof CM Rz — CQC (rad)	CQC · rad	2.5173×10^{-4}	2.52×10^{-4}	−0.108%	PASS
Roof CM Rz — SRSS (rad)	SRSS · rad	2.5173×10^{-4}	2.52×10^{-4}	−0.108%	PASS
Roof CM Rz — ABS (rad)	ABS · rad	2.5173×10^{-4}	2.52×10^{-4}	−0.108%	PASS
Axial elm 1 — CQC (kip)	CQC · kip	279.348	279.48	−0.047%	PASS
Axial elm 1 — SRSS (kip)	SRSS · kip	200.455	200.55	−0.047%	PASS
Axial elm 1 — ABS (kip)	ABS · kip	281.862	281.99	−0.045%	PASS
Axial elm 4 — CQC (kip)	CQC · kip	194.409	194.5	−0.047%	PASS
Axial elm 4 — SRSS (kip)	SRSS · kip	139.505	139.57	−0.047%	PASS
Axial elm 4 — ABS (kip)	ABS · kip	196.159	196.25	−0.046%	PASS
Axial elm 6 — CQC (kip)	CQC · kip	120.464	120.52	−0.046%	PASS
Axial elm 6 — SRSS (kip)	SRSS · kip	86.4431	86.48	−0.043%	PASS
Axial elm 6 — ABS (kip)	ABS · kip	121.548	121.61	−0.051%	PASS

§7 MODAL COMBINATION

METHOD	ROOF CM UX (IN)	DIFF
CQC	1.03241	−0.048%
SRSS	0.736835	−0.05%
ABS	1.04181	−0.047%

The reference publishes the roof centre-of-mass displacement under **three modal-combination rules**, and STRIX reproduces each: **CQC** and **SRSS** through the production RsaCalculator (Der Kiureghian CQC), and **ABS** (absolute sum) from the same per-mode responses. Because the two modes are **closely spaced** (frequencies within 2 %), CQC (`1.0329 in`) far exceeds SRSS (`0.7372 in`) — the cross-correlation adds the modes nearly in phase in X — while ABS (`1.0423 in`) is the upper bound. All three land `within tolerance` of the independent reference, verifying that STRIX handles closely-spaced modes correctly (the very case that motivates CQC).

§8 CONCLUSION

PASS

STRIX reproduces the **two modal frequencies**, the **roof centre-of-mass X/Y/Rz response**, and three **member axial forces** of the **CSI 1-025 / Peterson 1981** L-shaped braced frame under **all three combination rules** to **better than a quarter of a percent** across twenty independent quantities. Beyond SR2 (moment frame), this verifies three new surfaces at once: the **brace / axial-only member** in the modal and RSA path (linear Truss), a **rotational mass moment of inertia** reproduced through the de-Zhu diaphragm mass condensation (offset translational masses → exact Rz inertia), and a **tabulated spectrum with closely-spaced modes**. Together with SR1, SR2 and SM5/SM5b/SM6 it completes STRIX's response-spectrum verification across planar, eccentric-moment-frame and braced 3-D configurations.

§9 REFERENCES & ANALYSIS FILES

1. CSI SAP2000 Software Verification, Example 1-025 — Response-Spectrum Analysis of a Three-Dimensional Braced Frame.
2. F. E. Peterson (1981) — independent response-spectrum solution reproduced exactly by SAP2000.
3. E. L. Wilson, A. Der Kiureghian & E. P. Bayo, *EESD* **9**(2), 1981 — CQC rule. STRIX harness: `buildRsaPy → opensees.pyd (de-Zhu T-matrix, M_red = T^TMT + Transformation) → rsa_modal.bin`; MMI via four offset translational masses on the rigid diaphragm.

EVIDENCE ARCHIVE `verif-evidence-eng1.0.6-win-x64.zip · SR2b/` `sha256 (pending publish)`