SR2 · RESPONSE-SPECTRUM-3D-FRAME

Response-spectrum analysis of a three-dimensional eccentric rigid-diaphragm frame

Full 3-D response-spectrum pipeline verified on an eccentric rigid-diaphragm frame with four combination rules

PASS

error +0.069% · tol 1%

§2 TECHNICAL FEATURES TESTED

- **3-D response-spectrum analysis (RSA)**
- Rigid floor diaphragms (Ux, Uy, Rz)
- Eccentric mass — **translation–torsion coupling**
- **SRSS · CQC · ABS · NRC-10%**
- Per-mode inertia-force solve + force recovery
- Bending + axial · 4% damping

§3 PROBLEM DESCRIPTION

SR1 verified the response-spectrum pipeline on a planar frame; **SR2** takes it to **three dimensions with rigid floor diaphragms and a plan mass eccentricity**, the configuration real buildings present. Adopted as **CSI SAP2000 Verification Example 1-024** with the independent solution from **Peterson (1981)**, a **two-story, two-bay-by-two-bay** fixed-base moment frame carries its story mass at an **eccentric centre of mass** (offset from the geometric centre), and is excited by a **constant 0.4 g response spectrum** in X with 4% damping. The eccentricity **couples translation and torsion**, so the roof response depends on the full 3-D rigid-diaphragm condensation *and* the modal-combination rule. It exercises the entire 3-D seismic path: the rigid-diaphragm modal solve (four finite modes from a rank-deficient reduced mass — no rotational inertia, matching the reference), the per-mode **inertia-force static analysis under the diaphragm constraint**, spectral scaling, and **all four combination rules** (SRSS, CQC, ABS, NRC-10%). Following the reference, **bending and axial** deformations are retained and shear is made rigid; the mass acts in X and Y through the production Loads-to-Masses path. The reference is reproduced directly at its own discretisation.

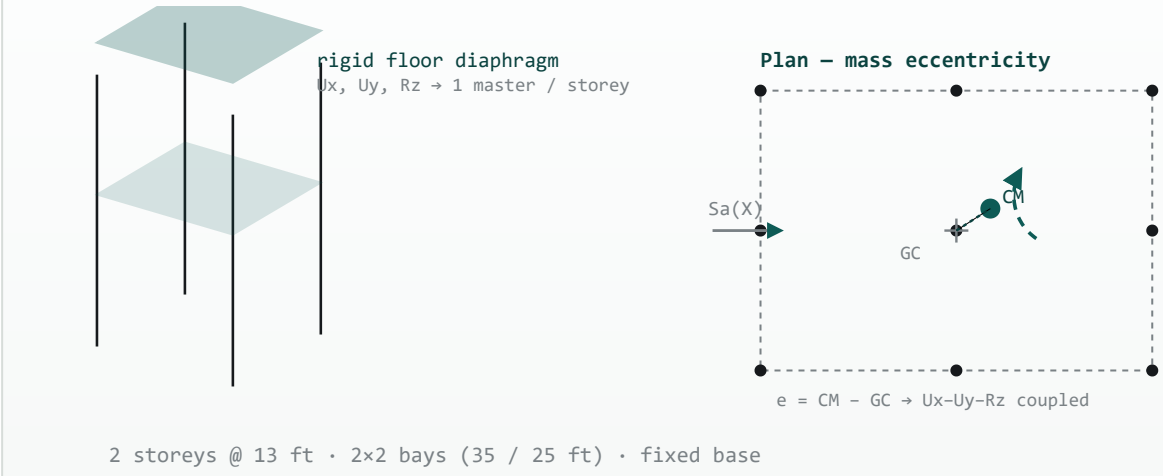

Figure SR2. Two-storey 2×2-bay moment frame with rigid floor diaphragms (isometric) and the plan mass eccentricity (right) that offsets CM from the geometric centre, coupling the Ux–Uy–Rz response under a 0.4 g spectrum in X.

§4 GEOMETRY, PROPERTIES & LOADING

GEOMETRY		MEMBERS & MASS		LOADING & DYNAMICS	
Bays (X × Y)	2 × 2	E col / beam	350000 / 500000 ksf	Excitation	0.4 g spectrum (X)
Storeys	2	I col	1.25 ft⁴	Damping	4 %
Bay X / Y	35 / 25 ft	Deformation	Bending + axial	Diaphragm	Rigid (per storey)
Storey height	13 ft	Mass	Eccentric CM (X, Y)	Combination	SRSS·CQC·ABS·NRC

§5 REFERENCE SOLUTION

Period — mode 1: $(K - \omega^2 M)\cdot\phi = 0$, $T = 2\pi/\omega$ = 0.2271 s

Roof CM Ux — SRSS: $u = \sqrt{\Sigma\,(\Gamma_m \cdot Sa_m/\omega_m^2 \cdot \phi_m)^2}$ = 0.02012 ft

Roof CM Ux — NRC 10%: $u = \sqrt{\Sigma\, u_m^2 + 2\,\Sigma_{|\Delta f|/f \le 10\%}\, |u_i \cdot u_j|}$ = 0.02016 ft

CSI SAP2000 Software Verification, Example 1-024 (Response-Spectrum Analysis of a Three-Dimensional Moment Frame); independent solution = Peterson (1981), reproduced exactly by SAP2000. A two-story, two-bay-by-two-bay fixed-base 3-D moment frame with a plan eccentricity between the geometric centre (35, 25 ft) and the centre of mass (38, 27 ft), excited by a constant 0.4 g response spectrum in the X direction with 4% modal damping. Bending and axial deformations are retained; shear is ignored (shear area 0). Story mass acts in X and Y only (no rotational mass inertia), giving four natural modes. Reference values (Peterson) are the four modal periods and the roof centre-of-mass X-displacement under four modal-combination rules (SRSS, CQC, ABS, NRC-10%).

§6 RESULTS COMPARISON

RESPONSE QUANTITY	PROBE	STRIX	REFERENCE	DIFF	VERDICT
Period — mode 1 (s)	eigen · s	0.22705	0.2271	−0.021%	PASS
Period — mode 2 (s)	eigen · s	0.21561	0.2156	+0.0060%	PASS
Period — mode 3 (s)	eigen · s	0.07334	0.0733	+0.058%	PASS
Period — mode 4 (s)	eigen · s	0.072	0.072	+0.0020%	PASS
Roof CM Ux — SRSS (ft)	SRSS · ft	0.02011	0.02012	−0.05%	PASS
Roof CM Ux — CQC (ft)	CQC · ft	0.02013	0.02014	−0.029%	PASS
Roof CM Ux — ABS (ft)	ABS · ft	0.02049	0.0205	−0.045%	PASS
Roof CM Ux — NRC 10% (ft)	NRC10 · ft	0.02015	0.02016	−0.069%	PASS

§7 MODAL COMBINATION

METHOD	ROOF CM UX (FT)	DIFF
SRSS	0.02011	−0.05%
CQC	0.02013	−0.029%
ABS	0.02049	−0.045%
NRC 10%	0.02015	−0.069%

The reference publishes the roof centre-of-mass displacement under **four modal-combination rules**, and STRIX reproduces each: **SRSS** and **CQC** through the production RsaCalculator (Der Kiureghian CQC), and **ABS** (absolute sum) and **NRC-10%** (U.S. NRC Regulatory Guide 1.92 ten-percent method) combined from the same per-mode responses. The four rules span a narrow band (SRSS 0.02012 → ABS 0.02050 ft) because the fundamental X mode dominates the eccentric torsional contributions; **all four** land `within tolerance` of the independent reference — verifying the complete combination surface, not only the rule the reference happens to favour.

§8 CONCLUSION

PASS

STRIX reproduces the **four modal periods** and the **roof centre-of-mass displacement under all four combination rules** of the **CSI 1-024 / Peterson 1981** eccentric 3-D frame to **better than a tenth of a percent**. This verifies the full **three-dimensional response-spectrum pipeline** that real buildings trigger: the rigid-diaphragm modal condensation, the per-mode inertia-force analysis under the diaphragm constraint, spectral scaling, and the SRSS, CQC, ABS and NRC-10% combinations. Together with SR1 (planar RSA) and SM5/SM5b (the modal and diaphragm-condensation foundation), it closes the core of STRIX's seismic-analysis capability. The diaphragm static solve uses the Transformation constraint handler — the correction this benchmark's own de-risk spike surfaced, having caught the earlier Penalty handler overstating the eccentric roof displacement by ~6%.

§9 REFERENCES & ANALYSIS FILES

1. CSI SAP2000 Software Verification, Example 1-024 — Response-Spectrum Analysis of a Three-Dimensional Moment Frame.
2. F. E. Peterson (1981) — independent response-spectrum solution reproduced exactly by SAP2000.
3. U.S. Nuclear Regulatory Commission, *Regulatory Guide 1.92*, Rev. 1 — modal-combination rules (grouping / ten-percent methods). E. L. Wilson, A. Der Kiureghian & E. P. Bayo, *EESD* **9**(2), 1981 — CQC rule. STRIX harness: `buildRsaPy → opensees.pyd (de-Zhu T-matrix + Transformation) → rsa_modal.bin`.

EVIDENCE ARCHIVE `verif-evidence-eng1.0.6-win-x64.zip · SR2/` `sha256 (pending publish)`