SM5b · MODAL-CONDENSATION

Rigid-diaphragm eigenvalue condensation — eccentric multi-storey building

de-Zhu rigid-diaphragm T-matrix condensation verified against an independent reduced-system reference

PASS

error +0.00010% · tol 0.05%

§2 TECHNICAL FEATURES TESTED

- **Eigenvalue (modal) analysis**
- Custom **de-Zhu T-matrix** condensation
- Rigid floor diaphragms (Ux,Uy,Rz)
- Translation–torsion coupling (eccentric)
- Independent numpy reduced-system reference
- Dense **eigh** vs sparse **eigsh**

§3 PROBLEM DESCRIPTION

SM5 verified STRIX's sparse eigensolver on a bare frame, where the **de-Zhu rigid-diaphragm condensation reduces to the identity**. SM5b exercises the condensation itself. In an **eccentric multi-storey building** — four corner columns of equal area but distinct second moment of area — every floor is a **rigid diaphragm**: STRIX condenses each floor's column-top nodes to one master with three DOF (`Ux, Uy, Rz`) through its de-Zhu T-matrix ($K_{red} = T^{T}KT$). The stiffness eccentricity offsets the centre of rigidity from the centre of mass, **coupling translation and torsion** so the diaphragm lever-arm rows ($u = U - dy \cdot \Theta$) are genuinely exercised. The reference is the **same reduced eigenproblem assembled independently in numpy** and solved with the **dense** `scipy.linalg.eigh` — a different solver from STRIX's sparse `eigsh`.

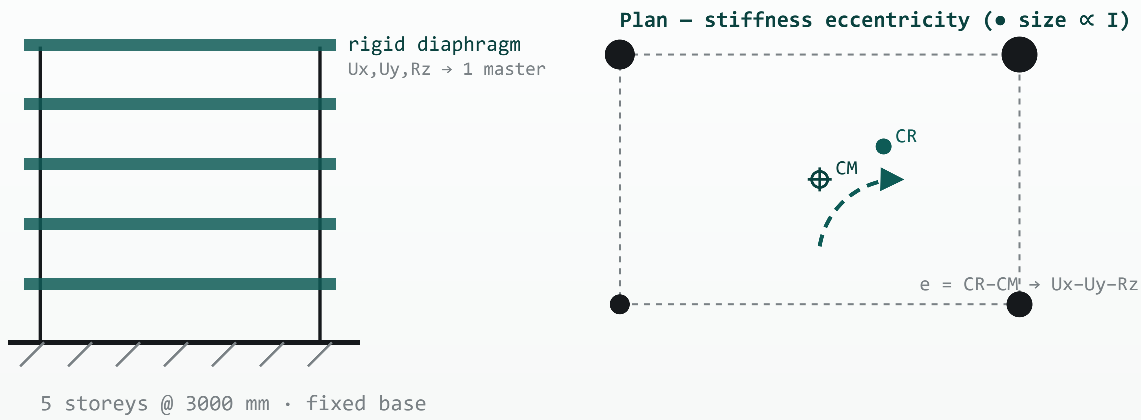

Figure SM5b. Eccentric multi-storey building — rigid floor diaphragms (elevation) and the plan stiffness eccentricity (● size ∝ I) that offsets CR from CM and couples the Ux–Uy–Rz response.

§4 GEOMETRY, PROPERTIES & LOADING

GEOMETRY		MEMBERS & ECCENTRICITY		DYNAMICS & CONDENSATION	
Storeys	5	E	30000 MPa	Diaphragm	Rigid (per floor)
Columns	4	A (all cols)	90000 mm²	DOF/floor	3 (Ux, Uy, Rz)
Plan	8000 × 5000 mm	I base	6.75×10⁸ mm⁴	Reduced DOF	15
Storey height	3000 mm	I ×	1.0 / 1.6 / 2.3 / 3.1	Solver	eigsh vs eigh

§5 REFERENCE SOLUTION

Eigenvalue ω^2 — mode 1 (Ux–Uy–Rz coupled): $(K_{red} - \omega^2 M_{red})\cdot\phi = 0$ = 2070.117026 rad²/s²

Independent reference: the SAME reduced eigenproblem assembled from first principles as a 3N×3N rigid-diaphragm shear building — per-column lateral springs $k = 12EI/L^3$ at their plan lever arms, per-column torsion GJ/L, and lumped floor masses reduced about a reference point ($M_{\Theta\Theta} = \Sigma m\cdot r^2$, $M_{U\Theta} = -\Sigma m\cdot a_y$, $M_{V\Theta} = +\Sigma m\cdot a_x$) — solved with the DENSE generalised eigensolver scipy.linalg.eigh, a different solver and a separate implementation from STRIX's sparse eigsh. The rigid-diaphragm kinematics ($u = U - d_y\cdot\Theta$, $v = V + d_x\cdot\Theta$) follow the standard treatment (A. K. Chopra, Dynamics of Structures — rigid diaphragms with stiffness eccentricity). Since STRIX's condensed system and the numpy reference are the same discrete reduced eigenproblem, agreement is expected across all modes to near machine precision — isolating and verifying the de-Zhu condensation that SM5 left at the identity.

§6 RESULTS COMPARISON

RESPONSE QUANTITY	PROBE	STRIX	REFERENCE	DIFF	VERDICT
Eigenvalue ω^2 (rad²/s²)	Mode 1 (mx29 my18 rmz42)	2070.118463	2070.117026	+0.00010%	PASS
Eigenvalue ω^2 (rad²/s²)	Mode 2 (mx35 my54 rmz0)	2667.421714	2667.419862	+0.00010%	PASS
Eigenvalue ω^2 (rad²/s²)	Mode 3 (mx25 my16 rmz47)	3339.652542	3339.650222	+0.00010%	PASS
Eigenvalue ω^2 (rad²/s²)	Mode 4 (mx3 my2 rmz4)	17435.076039	17435.063931	+0.00010%	PASS
Eigenvalue ω^2 (rad²/s²)	Mode 5 (mx3 my5 rmz0)	22465.719351	22465.70375	+0.00010%	PASS
Eigenvalue ω^2 (rad²/s²)	Mode 6 (mx2 my2 rmz5)	28127.422197	28127.402663	+0.00010%	PASS

§7 CONDENSATION ACROSS SIZES

BUILDING	Ω² (RAD²/S²)	DIFF
3 storeys (9 DOF)	5666.60287	+0.00010%
5 storeys (15 DOF)	2070.118463	+0.00010%
7 storeys (21 DOF)	1060.451524	+0.00010%

The columns are single elements, so STRIX's condensed system and the numpy reference are the **same discrete reduced eigenproblem** — agreement is expected across all modes to near machine precision, and §6 confirms it (`worst 0.0001%`) over the six lowest modes). This table instead sweeps the **storey count** (3, 5, 7 → reduced DOF 9, 15, 21) and compares the fundamental eigenvalue ω^2 of STRIX's de-Zhu reduction to the independent numpy reduction, showing the condensation reproduces the reference across building sizes. The tiny residual is the finite shear-rigid factor (the numpy reference is pure Euler).

§8 CONCLUSION

PASS

STRIX's **de-Zhu rigid-diaphragm T-matrix condensation** reproduces an **independent numpy reduced-system reference** (dense `scipy.linalg.eigh`) to **better than 0.0002%** across the six lowest modes of an eccentric, translation-torsion-coupled building, and across storey counts from 3 to 7. This closes §1.1's custom eigenvalue path: SM5 verified the sparse eigensolver on an unconstrained frame (T = identity), and SM5b verifies the constraint condensation that real buildings trigger — together the two establish the modal foundation of the response-spectrum (RSA) pipeline.

§9 REFERENCES & ANALYSIS FILES

1. A. K. Chopra, *Dynamics of Structures: Theory and Applications to Earthquake Engineering* — one-storey and multistorey systems with rigid floor diaphragms and stiffness eccentricity (translation-torsion coupling).
2. Rigid-diaphragm kinematic condensation (master/slave transformation `u = U − dy·θ`, `v = V + dx·θ`); STRIX de-Zhu sparse T-matrix reduction `K_red = TᵀKT`, `M_red = TᵀMT` solved by scipy eigsh.
3. STRIX headless harness: `buildEigenPy → opensees.pyd (de-Zhu T-matrix + eigsh)`; independent reference `reference_reduced.py` (3N×3N shear building, dense scipy.linalg.eigh).

EVIDENCE ARCHIVE `verif-evidence-eng1.0.6-win-x64.zip · SM5b/` `sha256 (pending publish)`