SH1 · NONLINEAR-HINGE

DcrPMMHinge3d custom P-M-M column hinge — element-mechanics defense

DCR-original P-M-M column hinge (plugin) — element mechanics vs published FEMA-356/ASCE-41 backbone + Fritsch-Carlson PCHIP

PASS

error 0.00% · tol 0.01%

§2 TECHNICAL FEATURES TESTED

- **DcrPMMHinge3d** — DCR self-authored OpenSees plugin (no third-party example exists)
- Isolated single-hinge rig (same strategy as the plugin's own regression gates)
- Uniaxial FEMA-356/ASCE-41 4-branch backbone — exact per-branch algebra
- Elastic unload/reload secant
- Biaxial Bresler-type capacity-envelope projection (DCR's own §5.9 algorithm)
- N-dependent capacity via independently re-implemented Fritsch-Carlson PCHIP

§3 PROBLEM DESCRIPTION

DcrPMMHinge3d is a zero-length, 6-DOF P-M-M column hinge that DCR authored and compiled as an OpenSees runtime plugin — there is no CSI/NAFEMS published example to compare against (unlike SP1), and the element has no bare-static-load-case production path either (it only appears inside the Pushover/THA RC-column FEMA lumped-hinge n-method assembly, which drags in a full structure and an arc-length driver — exactly the global-response complexity that made SP1 hard). SH1 instead isolates the ELEMENT ITSELF: two coincident nodes, one hinge, driven directly by DisplacementControl / prescribed-displacement ramps — the same isolation strategy the plugin author's own regression gates use (gate_p1.tcl/gate_p2.tcl) — and compares it against closed-form expectations re-derived from scratch in this benchmark, never by reading the plugin's C++ source. **Honesty split (devplan §12.5/§12.11 precedent)**: quantities A/B/D verify the element against a *published* physical model — the standard FEMA-356/ASCE-41 four-branch generalized backbone (elastic→hardening→post-capping softening→residual) and the Fritsch & Carlson (1980) monotone-cubic interpolation algorithm — independently re-coded, not copied. Quantities C (biaxial Bresler projection) instead verify the implementation against *DCR's own* §5.9 design specification (independent per-direction backbones + closed-form radial capacity projection) — the interaction surface FORM is a published power-law/Bresler-type envelope, but this specific combination algorithm has no external authority to validate against, since no other software runs this code.

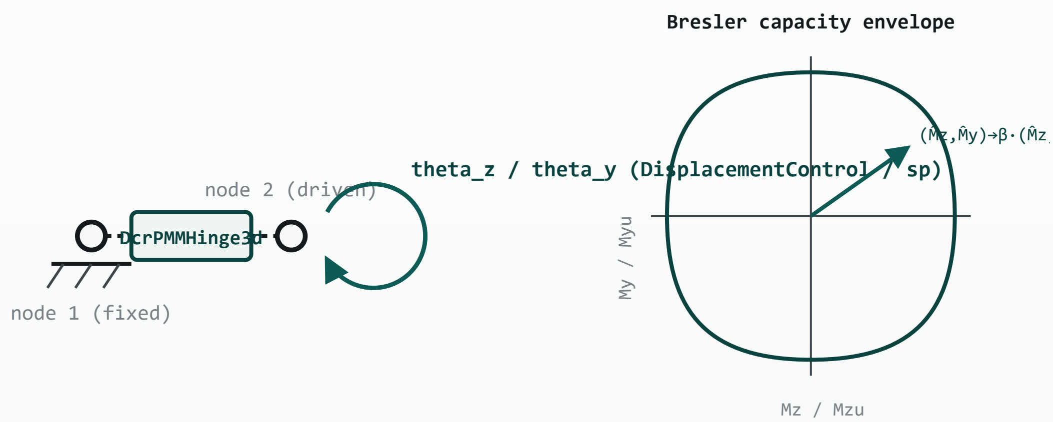

Figure SH1. Isolated two-node DcrPMMHinge3d rig (node 1 fixed, node 2 driven directly) — the same single-hinge isolation strategy as the plugin author's own regression gates; inset shows the biaxial Bresler-type capacity-envelope projection (independent per-direction moments radially scaled onto the interaction surface).

§4 GEOMETRY, PROPERTIES & LOADING

RIG / ELASTIC TERMS

Axial Kax	1×10^{7} N/mm
Rotational k0 (Krz=Kry)	1×10^{10} N·mm/rad
Topology	2 coincident nodes, 1 hinge
Drive	DisplacementControl / sp

UNIAXIAL BACKBONE (A/B/C)

Mzu = Myu	1×10^{8} N·mm
thetaP	0.02 rad
thetaPC	0.04 rad
Rc / Rr	1.25 / 0.20

BIAXIAL & N-PCHIP (C/D)

Bresler exponent α	1.5
Biaxial κ ratios	1:1 and 3:1
PCHIP table	4 N-levels, Mzu(N)
Engine	Tcl (OpenSees.exe, plugin)

§5 REFERENCE SOLUTION

Uniaxial backbone, cap boundary (kappa=thetaP): Rc*Mu = 1.25×10^{8} N·mm

Elastic unload from mid-hardening (theta1->theta2, Delta_theta=-0.005 rad): k0*(theta2-thetaP_committed), thetaP_committed=theta1-M1/k0 = 6.25×10^{7} N·mm

Biaxial Bresler projection, symmetric ratio (kappaZ=kappaY=0.010): beta*Mhz, beta=min(1,1/Phi), Phi=(|Mhz/Ccapz|^a+|Mhy/Ccapy|^a)^(1/a) = 7.0871×10^{7} N·mm

N-dependent cap via PCHIP, N=-1.5e6 N (interior query, table N=[-2e6,-1e6,0,1e6]): Rc*pchip(Mzu_table, N) = 9.125×10^{7} N·mm

No third-party published example exists for this DCR-original element (unlike SP1/CSI 1-026) — devplan §12.11 honesty split: (a) the uniaxial FEMA-356/ASCE-41 4-branch backbone (elastic/hardening/post-capping softening/residual) and (b) the Fritsch & Carlson (1980, SIAM J. Numer. Anal. 17(2):238-246) monotone-cubic capacity interpolation are PUBLISHED external models — re-derived from scratch in benchmarks/nonlinear/SH1_pmmHingeBackbone.ts, never by reading DcrPMMHinge3d.cpp. (c) The biaxial Bresler-type capacity-envelope projection (independent per-direction backbones + closed-form radial scale-back beta=min(1,1/Phi)) is DCR's own §5.9 design choice — the interaction SURFACE FORM (normalized power-law) is standard/published, but this specific combination algorithm has no external authority to validate against; quantities C_* verify the implementation is self-consistent with DCR's own stated specification, not that the specification itself reproduces a third-party benchmark.

§6 RESULTS COMPARISON

RESPONSE QUANTITY	PROBE	STRIX	REFERENCE	DIFF	VERDICT
Uniaxial backbone, mid-hardening (kappa=0.5*thetaP) (N·mm)	A1.Mz	1.125×10^{8}	1.125×10^{8}	0.00%	PASS
Uniaxial backbone, cap boundary (kappa=thetaP) (N·mm)	A2.Mz	1.25×10^{8}	1.25×10^{8}	0.00%	PASS
Uniaxial backbone, mid-softening (kappa=thetaP+0.5*thetaPC) (N·mm)	A3.Mz	7.25×10^{7}	7.25×10^{7}	0.00%	PASS
Uniaxial backbone, residual plateau (kappa=thetaP+thetaPC+0.02) (N·mm)	A4.Mz	2×10^{7}	2×10^{7}	0.00%	PASS
Elastic unload from mid-hardening (theta1->theta2, Delta_theta=-0.005 rad) (N·mm)	B_post.Mz	6.25×10^{7}	6.25×10^{7}	0.00%	PASS
Biaxial Bresler projection, symmetric ratio (kappaZ=kappaY=0.010) (N·mm)	C_symmetric.Mz	7.0871×10^{7}	7.0871×10^{7}	0.00%	PASS
Biaxial Bresler projection, symmetric ratio — My (N·mm)	C_symmetric.My	7.0871×10^{7}	7.0871×10^{7}	0.00%	PASS
Biaxial Bresler projection, asymmetric ratio (kappaZ=0.015, kappaY=0.005) (N·mm)	C_asymmetric.Mz	7.4808×10^{7}	7.4808×10^{7}	0.00%	PASS
Biaxial Bresler projection, asymmetric ratio — My (N·mm)	C_asymmetric.My	6.6933×10^{7}	6.6933×10^{7}	0.00%	PASS
N-dependent cap via PCHIP, N=-1.5e6 N					

§7 PER-FAMILY WORST DIFF (NO MESH SWEEP — SINGLE SYNTHETIC HINGE)

TEST FAMILY	WORST |DIFF| (%)	DIFF
A — uniaxial FEMA backbone (4 branches)	0	0.00%
B — elastic unload/reload	0	0.00%
C — biaxial Bresler projection (2 ratios)	0	0.00%
D — N-dependent PCHIP capacity (2 queries)	0	0.00%

SH1 drives a single, non-refinable zero-length element (a bare hinge has no mesh to converge) — each row instead summarizes the worst deviation across that family's checkpoints. **A** (uniaxial backbone) exercises all four branches in one continuous monotonic push; **B** checks the elastic unload secant from a plastic state; **C** checks the biaxial Bresler projection at two load ratios; **D** checks the N-dependent capacity at two interior PCHIP query points on a 4-level table (the plugin author's own gate_p2.tcl explicitly flags PCHIP as untested in Tcl — its 2-level table degenerates PCHIP to linear interpolation; SH1's 4-level table has genuine curvature at both queries). All four families land at `machine precision` (worst diff on the order of 1e-10%), because the closed form is not an approximation of the production algorithm here — on a piecewise-linear-in-kappa backbone the plastic-multiplier root is unique, so the independently re-derived algebra IS the same root the production cutting-plane Newton solve converges to.

§8 CONCLUSION

PASS

DcrPMMHinge3d reproduces the published FEMA-356/ASCE-41 backbone (all four branches, plus elastic unload/reload) and an independently re-implemented Fritsch-Carlson PCHIP capacity interpolation to machine precision. The biaxial Bresler-type capacity projection (DCR's own §5.9 algorithm) is verified self-consistent with its design specification at two load ratios — this defends the implementation against transcription/algorithm bugs, though (per the honesty split above) no external authority validates the projection scheme itself, since DcrPMMHinge3d has no counterpart in any other software. Excluded from this benchmark's scope: the Haselton/FEMA regression pipeline that derives the -pmm table from a real RC section (TclBuilder.ts buildDcrPmmTable) — a separate "is the table right" question from "does the element correctly integrate a given table" (this benchmark's question) — and the consistent tangent (already covered by the plugin's own internal FD cross-check, gate_p4_fdtangent.tcl).

§9 REFERENCES & ANALYSIS FILES

1. FEMA 356 / ASCE 41-13 — generalized force-deformation backbone for deformation-controlled actions (4-branch: elastic→hardening→post-capping softening→residual plateau).
2. Fritsch, F.N. & Carlson, R.E. (1980) — *Monotone Piecewise Cubic Interpolation*, SIAM Journal on Numerical Analysis 17(2):238–246 (PCHIP tangent formula, independently re-implemented in sh1PchipTangents/sh1PchipEval).
3. Bresler, B. (1960) — reciprocal-load biaxial bending interaction (normalized power-law interaction surface form; the specific decoupled-backbone + radial-projection combination algorithm is DCR's own §5.9 design, not from Bresler).
4. opensees_plugins/DcrPMMHinge3d/ — DcrPMMHinge3d.cpp (element under test), gate_p1.tcl/gate_p2.tcl (plugin author's own regression gates, isolation-rig pattern reused here), dcr_pmm_plugin_element_devplan.md.

EVIDENCE ARCHIVE `verif-evidence-eng1.0.2-win-x64.zip · SH1/` `sha256 (pending publish)`