SB9 · STATIC-FRAME

Rigid steel portal frame under UDL — bending + axial deformation (CSI 1-018)

PASS

Rigid steel portal frame under UDL — bending + axial deformation vs unit-load method

error +0.0106% · tol 0.05%

§2 TECHNICAL FEATURES TESTED

- **elasticBeamColumn** — Euler-Bernoulli (no shear area, TclBuilder.ts:465)
- **endReleases** (My) — pinned column-beam joints, production penalty release
- **Section.stiffScale** — CSI-style property modification factor (area/izz)
- Distributed load (`eleLoad -type -beamUniform`)
- Deflection vs **unit-load / virtual-work** method (Cook & Young 1985)

§3 PROBLEM DESCRIPTION

A one-story, one-bay rigid steel bent (W8X31) carries a uniform vertical load across its beam. The base supports and the two beam-to-column joints are all pins, so the frame reduces to a determinate system for this symmetric vertical load: each column behaves as a pure two-force (axial-only) member, and the beam behaves as an ordinary simply-supported beam under UDL. STRIX reproduces the pinned joints with production `endReleases` (My = free at each column top) and isolates the bending and axial contributions separately using `Section.stiffScale` — the same "property modification factor" technique CSI SAP2000 uses in this exact verification example. Because `elasticBeamColumn` carries no shear-area term (Euler-Bernoulli only, same scope as SB1), the shear-deformation sub-model from CSI 1-018 is out of scope; the bending-only and axial-only STRIX runs sum to the combined-model result, matching the additive structure of the unit-load integral itself.

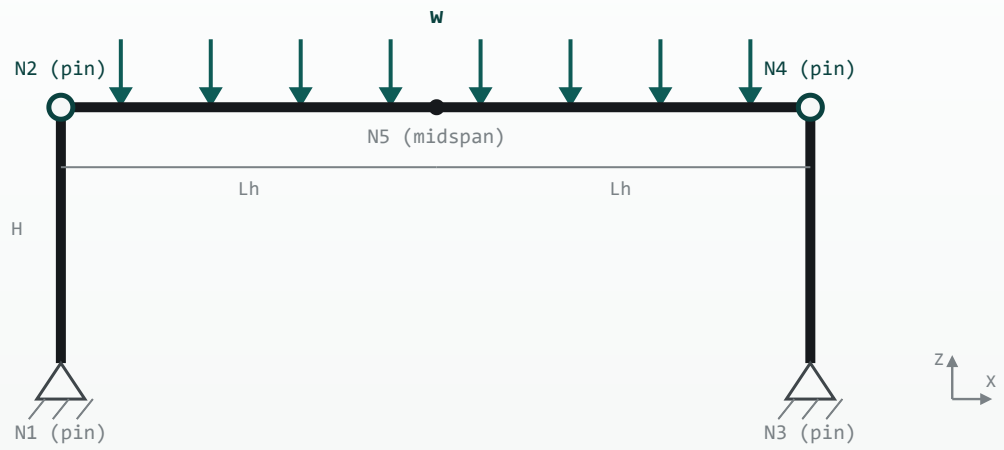

Figure SB9. Rigid steel portal frame: base pins (N1, N3), pinned column-beam joints (N2, N4), UDL w across the beam.

§4 GEOMETRY, PROPERTIES & LOADING

GEOMETRY		MATERIAL & SECTION		LOADING & BC	
Column height H	144 in	E	29900 ksi	Supports	4 pins (base ×2, joint ×2)
Beam half-span Lh	144 in	ν	0.3	Load w	0.1 kip/in
Full span S	288 in	Area A	9.12 in²	Location	full beam span
Section	W8X31	Strong-axis Iz	110 in⁴	Self-weight	off
				Analysis	Linear static

§5 REFERENCE SOLUTION

Midspan deflection Uz — combined (axial+bending): dTotal = V*H/(E*A) + 5*w*S^4/(384*E*Iz) (unit-load method, no shear term) = -69.372833 mm

Midspan deflection Uz — bending only (area x1e5): dBending = 5*w*S^4/(384*E*Iz) (simply-supported UDL beam, midspan) = -69.179684 mm

Midspan deflection Uz — axial only (Iz x1e7): dAxial = V*H/(E*A) (pin-pin column, 2-force member) = -0.193149 mm

CSI SAP2000 Software Verification Example 1-018 'Bending, Shear and Axial Deformations in a Rigid Frame'; independent reference = unit-load (virtual-work) method, Cook & Young, Advanced Mechanics of Materials, 1985, p.244. STRIX omits the shear term (elasticBeamColumn has no shear-area argument, TclBuilder.ts:465 — Euler-Bernoulli only, same scope as SB1) and re-derives the axial + bending terms from scratch via sb9Theory(); the result matches CSI's published in-lb values (converted to N·mm) to 6 significant figures: axial 0.0076043 vs CSI 0.00760 in, bending 2.7236096 vs CSI 2.723610 in.

§6 RESULTS COMPARISON

RESPONSE QUANTITY	PROBE	STRIX	REFERENCE	DIFF	VERDICT
Midspan deflection Uz — combined (axial+bending) (mm)	N5.uz	-69.3655	-69.372833	+0.0106%	PASS
Midspan deflection Uz — bending only (area x1e5) (mm)	N5.uz	-69.1723	-69.179684	+0.0107%	PASS
Midspan deflection Uz — axial only (Iz x1e7) (mm)	N5.uz	-0.193176	-0.193149	-0.0139%	PASS

§7 DEFORMATION-MODE BREAKDOWN

MODEL (STIFFSCALE)	MIDSPAN UZ (MM)	DIFF
combined (A, Iz normal)	-69.3655	+0.0106%
bendingOnly (A x1e5)	-69.1723	+0.0107%
axialOnly (Iz x1e7)	-0.193176	-0.0139%

Rather than a mesh sweep (this is a single-discretization determinate/closed-form problem, not a convergence study), the three rows isolate each deformation mode using CSI's own "property modification factor" technique: `bendingOnly` inflates the section area $\times 10^5$ to suppress axial deformation, and `axialOnly` inflates the strong-axis inertia $\times 10^7$ to suppress bending. All three reproduce the unit-load reference to `under 0.02%` — the small residual comes from the production `endReleases` penalty pin (a finite, not exactly-zero, release stiffness), which is itself the real production code path being verified.

§8 CONCLUSION

PASS STRIX reproduces the reference solution to **+0.0106%** (tolerance 0.05%). The tested element and its formulation are verified against the reference.

§9 REFERENCES & ANALYSIS FILES

1. Cook, R. D. & Young, W. C. — *Advanced Mechanics of Materials.* Macmillan, 1985. Unit-load (virtual-work) method, p.244.
2. CSI — *SAP2000 Software Verification, Example 1-018:* Bending, Shear and Axial Deformations in a Rigid Frame (independent reference method + property modification factors).
3. AISC — W8X31 section properties (A, Iz, Iy, J).
4. OpenSees — `element elasticBeamColumn` (Euler-Bernoulli); STRIX `endReleases` (penalty pin) + `Section.stiffScale`.

EVIDENCE ARCHIVE `verif-evidence-eng1.0.2-win-x64.zip · SB9/` `sha256 (pending publish)`