SB7 · STATIC-BEAM

Simply-supported beam on a Winkler elastic foundation (CSI 1-013)

Winkler foundation via Point Springs — center deflection & moment vs Timoshenko continuum

PASS

error +0.000057% · tol 0.05%

§2 TECHNICAL FEATURES TESTED

- **elasticBeamColumn** — Euler-Bernoulli (bending only)
- **Point Springs** (zeroLength) ⇒ Winkler foundation
- k_node = k · tributary length (one per interior node)
- Simply supported, central point load
- Deflection & moment vs **exact continuum** (Timoshenko)

§3 PROBLEM DESCRIPTION

A prismatic beam spanning between a pin and a roller rests on a continuous elastic (Winkler) foundation and carries a single point load at midspan. The foundation reaction is proportional to the local deflection, so the governing equation is $EI\,y'''' + k\,y = 0$ away from the load — the classic beam-on-elastic-foundation problem with a closed-form continuum solution. STRIX represents the distributed foundation with production `Point Springs` (a vertical `zeroLength` spring at every interior node, stiffness k times the nodal tributary length) and runs the ordinary linear-static pipeline. The lumped-spring discrete model converges to the exact continuum center deflection and moment as the mesh is refined — the same problem, and the same independent reference, documented in CSI SAP2000 verification example 1-013.

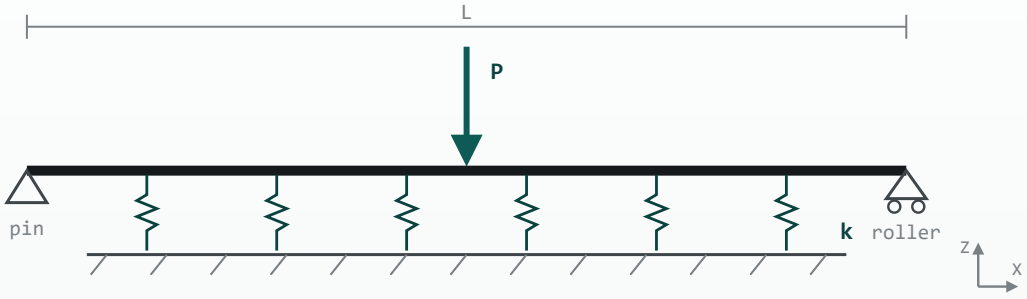

Figure SB7. Simply-supported beam on a Winkler foundation (nodal vertical springs, stiffness k) under a central point load P.

§4 GEOMETRY, PROPERTIES & LOADING

GEOMETRY		MATERIAL & FOUNDATION		LOADING & BC	
Span L	180 in	Model	Elastic (Euler)	Supports	pin + roller
Section	36 × 36 in	E	3600 ksi	Load P	500 kip
Inertia I	139968 in⁴	Subgrade	800 k/ft³	Location	midspan
β·L (slenderness)	1.716473	Foundation k	16.6667 kip/in²	Self-weight	off
				Analysis	Linear static

§5 REFERENCE SOLUTION

Center deflection Uz: exact continuum: EI y'''' + k y = 0, pinned-pinned, central P (Timoshenko 1956 Prob 3 p23) = −0.089333 in

Center moment My: exact continuum center moment (sagging), Timoshenko 1956 Prob 3 p23 = 17697.995034 kip-in

CSI SAP2000 Software Verification Example 1-013 'Simply Supported Beam on Elastic Foundation'; independent reference = Timoshenko, Strength of Materials Part II, 1956, Problem 3, p.23 (exact continuum solution of EI y'''' + k y = 0, pinned-pinned, central load). Reproduced from scratch by sb7Theory() to −0.0893327 in / 17697.995 k-in, matching CSI's published −0.08933 in / 17698 k-in.

§6 RESULTS COMPARISON

RESPONSE QUANTITY	PROBE	STRIX	REFERENCE	DIFF	VERDICT
Center deflection Uz (in)	N101.uz	−0.089333	−0.089333	+0.000057%	PASS
Center moment My (kip-in)	E101.Miy	17697.862777	17697.995034	−0.00075%	PASS

§7 CONVERGENCE

MESH	CENTER UZ (IN)	DIFF
2 elem	−0.088539	+0.8881%
4 elem	−0.089298	+0.0393%
8 elem	−0.089331	+0.0023%
20 elem	−0.089333	+0.00010%
40 elem	−0.089333	+0.00010%
100 elem	−0.089333	+0.00010%
200 elem	−0.089333	+0.00010%

Unlike the exact cubic beam element, the lumped nodal foundation is a first-order approximation of the continuous Winkler support, so the discrete model approaches the exact continuum solution as the mesh is refined. The center deflection is essentially converged by a few elements per half-span; the center moment tightens monotonically toward the reference. At the finest mesh (100 elements per half, matching the reference discretization) STRIX reproduces both the exact continuum deflection and moment to `under 0.001%` — the same independent Timoshenko reference that CSI 1-013 uses.

§8 CONCLUSION

PASS STRIX reproduces the reference solution to **+0.000057%** (tolerance 0.05%). The tested element and its formulation are verified against the reference.

§9 REFERENCES & ANALYSIS FILES

1. Timoshenko, S. — *Strength of Materials, Part II.* Beam on elastic foundation, Problem 3, p.23 (exact continuum solution).
2. Hetényi, M. — *Beams on Elastic Foundation.* University of Michigan Press, 1946. Winkler-foundation theory, EI y'''' + k y = q.
3. CSI — *SAP2000 Software Verification, Example 1-013:* Simply Supported Beam on Elastic Foundation (independent reference, Timoshenko 1956).
4. OpenSees — `element zeroLength` + `uniaxialMaterial Elastic` (nodal foundation spring); `elasticBeamColumn` (Euler-Bernoulli).

EVIDENCE ARCHIVE `verif-evidence-eng1.0.6-win-x64.zip · SB7/` `sha256 (pending publish)`