SB5 · STATIC-PLATE

Rectangular plate bending — thin plate, uniform & central point load

Plate bending — production ShellMITC4 slab vs Timoshenko thin-plate theory

PASS

error −0.621% · tol 2%

§2 TECHNICAL FEATURES TESTED

- **ShellMITC4** — production flat-plate slab element
- ElasticMembranePlateSection (Mindlin plate)
- Simply-supported & clamped edges
- Uniform pressure + central point load
- 1:1 and 5:1 aspect ratios (8 cases)
- h-refinement convergence to closed form

§3 PROBLEM DESCRIPTION

A rectangular plate carries an out-of-plane load and its center deflection is compared to the **Timoshenko–Woinowsky thin-plate series solution**. Following **CSI SAP2000 Verification Example 2-005** (geometry after MacNeal–Harder), eight configurations are run — a square (1:1) and a 5:1 rectangular plate, each **simply-supported** and **clamped**, under a **uniform pressure** and a **central point load**. STRIX recovers the dimensionless deflection coefficient $\alpha = w \cdot D/(q \cdot a^4)$ (uniform) or $w \cdot D/(P \cdot a^2)$ (point), which is scale-invariant and compared to the published table value. The plate is a **production flat-plate slab**, meshed with the exact `ShellMITC4` + `ElasticMembranePlateSection` element a real STRIX slab uses. It is kept thin ($a/t = 200$) so transverse shear is negligible and this Mindlin element converges to the thin-plate coefficient (the thick-plate regime is covered separately). At structural scale the automatic rotational stabiliser is negligible and is removed.

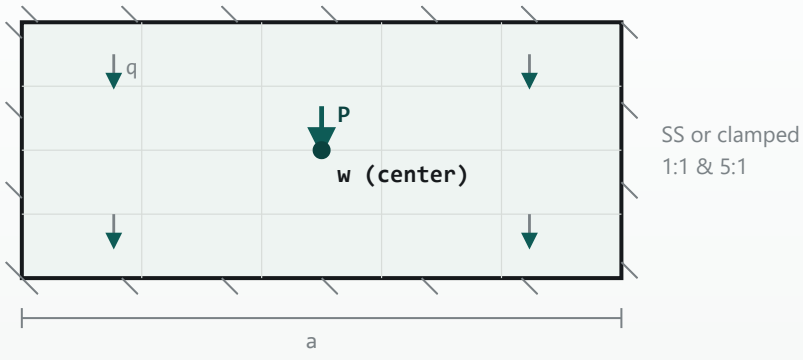

Figure SB5. Rectangular plate (1:1 shown; 5:1 also tested) — supported edges, uniform pressure q and central point load P, monitored by the center deflection w.

§4 GEOMETRY, PROPERTIES & LOADING

GEOMETRY		MATERIAL		LOADING & BC	
Width a	2000 mm	Model	Elastic · thin plate	Uniform q	1×10^{-4} N/mm²
Aspects	1:1 & 5:1	E	30000 MPa	Point P	100 N
Thickness t	10 mm	ν	0.3	Supports	SS & clamped
a / t	200	Units	N · mm	Probe	center defl. w

§5 REFERENCE SOLUTION

α — SS, 1:1, uniform: $w = \alpha \cdot q \cdot a^4/D$ (Table 8) = 0.00406

α — SS, 1:1, point: $w = \alpha \cdot P \cdot a^2/D$ (Eq.147, Table 23) = 0.0116

Timoshenko & Woinowsky-Krieger, Theory of Plates and Shells, 2nd ed. (1959): Table 8 (SS, uniform), Eq.147 & Table 23 (SS, central point), Table 35 (clamped, uniform), Table 37 (clamped, central point), ν = 0.3. The eight-case layout follows CSI SAP2000 Verification Example 2-005 (geometry after MacNeal & Harder, 1985). STRIX recovers the scale-invariant deflection coefficient α (= $w \cdot D/(q \cdot a^4)$ uniform, $w \cdot D/(P \cdot a^2)$ point) and compares it to the published table value; a = short side.

§6 RESULTS COMPARISON

RESPONSE QUANTITY	PROBE	STRIX	REFERENCE	DIFF	VERDICT
Deflection coefficient α	SS · 1:1 · uniform	0.00406	0.00406	+0.064%	PASS
Deflection coefficient α	SS · 5:1 · uniform	0.01294	0.01297	−0.264%	PASS
Deflection coefficient α	Fixed · 1:1 · uniform	0.00126	0.00126	+0.336%	PASS
Deflection coefficient α	Fixed · 5:1 · uniform	0.00259	0.0026	−0.506%	PASS
Deflection coefficient α	SS · 1:1 · central point	0.0116	0.0116	−0.0020%	PASS
Deflection coefficient α	SS · 5:1 · central point	0.01694	0.01695	−0.069%	PASS
Deflection coefficient α	Fixed · 1:1 · central point	0.00559	0.0056	−0.165%	PASS
Deflection coefficient α	Fixed · 5:1 · central point	0.00721	0.00725	−0.621%	PASS

§7 CONVERGENCE

MESH	ALPHA	DIFF
8×40	0.00689	−4.944%
16×80	0.00715	−1.329%
24×120	0.00721	−0.621%

For every case the first-order MITC4 plate approaches the Timoshenko coefficient `monotonically from below` (a coarse mesh is slightly stiff), reaching the closed form to a fraction of a percent at the finest mesh — comfortably better than the commercial verification tolerance for the same problem. §7 shows the governing (worst final) case; all eight appear at the finest mesh in §6. The plate is thin, so the residual transverse-shear of this Mindlin element is under 0.1%; the auto rotational stabiliser is stripped and was measured to shift the answer by under 0.2% even when present.

§8 CONCLUSION

PASS

Across all eight CSI 2-005 configurations — two aspect ratios, simply-supported and clamped, uniform and central point load — the recovered deflection coefficient matches the Timoshenko–Woinowsky thin-plate solution to well within tolerance, converging monotonically. The production slab element **ShellMITC4 + ElasticMembranePlateSection** is verified for out-of-plane plate bending under both distributed and concentrated load and both support conditions. Thick-plate (transverse-shear) behaviour is verified separately against Roark.

§9 REFERENCES & ANALYSIS FILES

1. S. Timoshenko & S. Woinowsky-Krieger, *Theory of Plates and Shells*, 2nd ed. (1959) — Tables 8, 23, 35, 37 & Eq.147 (ν = 0.3); see §5.
2. CSI SAP2000 Software Verification, Example 2-005 — Rectangular Plate with Static Loads (geometry after MacNeal & Harder, 1985); same eight-case layout, center-deflection check.
3. STRIX headless harness: `production flat_plate slab → generatePyForLoadCase → opensees.pyd`; ShellMITC4 + ElasticMembranePlateSection, auto rotational stabiliser removed.

EVIDENCE ARCHIVE `verif-evidence-eng1.0.6-win-x64.zip · SB5/` `sha256 (pending publish)`