SB3 · STATIC-MEMBRANE

Cook's membrane (skew panel, in-plane bending on a distorted mesh)

In-plane bending on a distorted mesh — custom membrane robustness check

PASS

error +0.201% · tol 3%

§2 TECHNICAL FEATURES TESTED

- **ASDShellQ4DCR** — custom in-plane shell
- Membrane subType (production default)
- Skewed (trapezoidal) mesh — distortion test
- Combined in-plane bending + shear
- h-refinement convergence study

§3 PROBLEM DESCRIPTION

Cook's membrane is the standard distortion benchmark for in-plane elements: a tapered panel is **clamped on its left edge** A–D and loaded by a uniform shear traction on its right edge B–C, producing a combined bending–shear response. A uniform mesh of the trapezoid gives **skewed (non-rectangular) elements**, so how quickly the answer is recovered measures the element's robustness to mesh distortion. The monitored quantity is the vertical displacement at the loaded-edge midpoint Q, reported as the scale-independent normalized value $u_Q \cdot E \cdot t/P$ (the problem has no closed form; the reference is the plane-stress converged value). The panel is modelled at real structural scale so the element's auto drilling stabiliser is physically negligible, as it is in production.

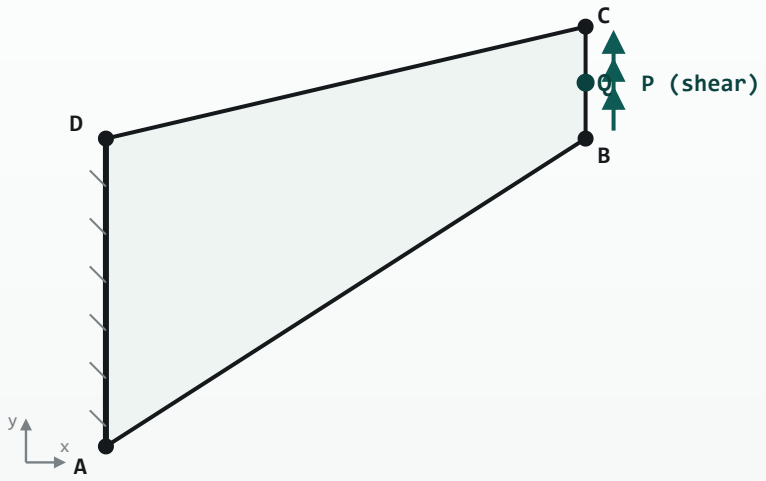

Figure SB3. Cook's skew panel — clamped edge A–D, uniform shear on B–C, monitored at the loaded-edge midpoint Q.

§4 GEOMETRY, PROPERTIES & LOADING

GEOMETRY		MATERIAL		LOADING & BC	
Corners (unit)	A0,0 B48,44 C48,60 D0,44	Model	Elastic · plane stress	Shear on B–C	1×10⁶ N (+Y)
Scale	100 × (mm)	E	30000 MPa	Clamp	u_x=u_y=0 (A–D)
Probe Q	mid edge (48,52)	ν	0.333333	Out-of-plane	u_z, r_x, r_y fixed
Thickness t	200 mm	Units	N · mm	Analysis	Linear static

§5 REFERENCE SOLUTION

Normalized tip displacement u_Q*E*t/P (mid loaded edge): u_y(Q)*E*t/P, Q = midpoint of loaded edge (plane stress, nu=1/3) = 23.91 norm.

*Cook's membrane, a standard finite-element distortion benchmark (R.D. Cook, 1974). No closed-form solution; the reference is the fine-mesh converged tip displacement at the loaded-edge midpoint Q, PLANE STRESS, E=1, nu=1/3, unit shear resultant: normalized u_Q*E*t/P = 23.91 (literature band 23.90-23.96). The plane-strain counterpart is 21.52 (Sevilla et al. / locking-free FCFV report 21.42 converged); DCR walls are plane stress (membrane section), and 21.42/(1-nu^2) ~= 23.9 confirms the plane-stress mid-edge value.*

§6 RESULTS COMPARISON

RESPONSE QUANTITY	PROBE	STRIX	REFERENCE	DIFF	VERDICT
Normalized tip displacement u_Q*E*t/P (mid loaded edge) (norm.)	mesh 64×64	23.9579	23.91	+0.201%	PASS

§7 CONVERGENCE

MESH	U_Q*E*T/P (NORM.)	DIFF
2×2 cells	20.7346	−13.281%
4×4 cells	22.9927	−3.836%
8×8 cells	23.6869	−0.933%
16×16 cells	23.8831	−0.113%
32×32 cells	23.9397	+0.124%
64×64 cells	23.9579	+0.201%

The bilinear ASDShellQ4DCR membrane converges to the plane-stress reference `monotonically from below` on the distorted trapezoidal mesh, reaching the reference to a fraction of a percent at the finest mesh. Even the very coarse meshes are accurate — the enhanced-strain formulation recovers most of the answer where a plain bilinear element would lag badly under this distortion. The drilling degree of freedom is essential to this bending accuracy, and the auto stabiliser spring is negligible at the structural scale used, so the converged value is a property of the element, not of the stabiliser.

§8 CONCLUSION

PASS

On Cook's distorted mesh the normalized mid-edge displacement $u_Q \cdot E \cdot t/P$ rises monotonically toward the plane-stress reference of 23.91 and the finest mesh lies well within tolerance, with strong coarse-mesh accuracy. The custom in-plane shell **ASDShellQ4DCR** is verified to be robust to mesh distortion in combined in-plane bending and shear — and the drilling stabiliser is confirmed non-polluting for in-plane bending, complementing the P3-S2 modal study.

§9 REFERENCES & ANALYSIS FILES

1. R.D. Cook (1974) — skew cantilever ("Cook's membrane"), the standard mesh-distortion benchmark. No closed-form solution.
2. Plane-stress reference (ν=1/3, unit shear): normalized $u_Q \cdot E \cdot t/P \approx 23.90–23.96$ at the loaded-edge midpoint. Plane-strain counterpart 21.52 (Sevilla et al. / FCFV report 21.42 converged); $21.42/(1-\nu^2) \approx 23.9$ fixes the plane-stress value.
3. STRIX headless harness: `generatePyForLoadCase → opensees.pyd → node_disp.out`, production-default membrane wall.

EVIDENCE ARCHIVE `verif-evidence-eng1.0.6-win-x64.zip · SB3/` `sha256 (pending publish)`