SB2 · STATIC-MEMBRANE

NAFEMS LE1 elliptic membrane (tangential edge stress at D)

In-plane membrane stress at a stress raiser — custom shell front-line check

PASS

error −1.994% · tol 3%

§2 TECHNICAL FEATURES TESTED

- **ASDShellQ4DCR** — custom in-plane shell
- Membrane subType: **PlateFromPlaneStress** + PlateFiber
- Drilling stabiliser + rotFloor active
- Gauss-point membrane stress resultants
- h-refinement convergence study

§3 PROBLEM DESCRIPTION

A quarter of an elliptic annulus is loaded by a uniform outward normal tension on its outer elliptic edge B–C, with symmetry restraints on the two straight edges. The tangential edge stress σ_{yy} at the inner point D is a classic stress-raiser and the NAFEMS acceptance quantity. It is recovered as the maximum principal membrane stress at the Gauss point nearest D (at D the symmetry axis and the free inner edge make σ_{yy} a principal stress, so the value is frame-invariant). First-order quadrilaterals approach the reference from below, so the element is exercised over a mesh-refinement sequence.

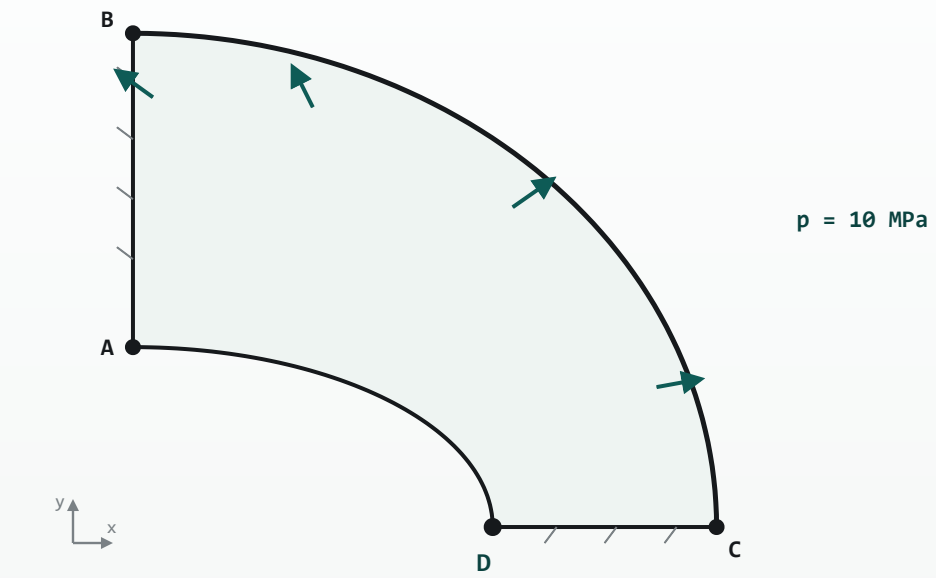

Figure SB2. Quarter elliptic membrane — outer arc B–C under outward tension, symmetry on the straight edges, evaluation point D.

§4 GEOMETRY, PROPERTIES & LOADING

GEOMETRY		MATERIAL		LOADING & BC	
Inner semi-axes	2000 × 1000 mm	Model	Elastic · plane stress	Outer arc B–C	10 MPa (tension)
Outer semi-axes	3250 × 2750 mm	E	210000 MPa	Symmetry	u_x=0 (AB), u_y=0 (CD)
Point D	2000, 0 mm	ν	0.3	Out-of-plane	u_z, r_x, r_y fixed
Thickness t	100 mm	Units	N · mm	Analysis	Linear static

§5 REFERENCE SOLUTION

Tangential edge stress sigma_yy at D: sigma_yy(D) = max principal of (N11,N22,N12)/t at the Gauss point nearest D = 92.7 MPa

NAFEMS, The Standard NAFEMS Benchmarks (Linear Elastic), Benchmark LE1 — elliptic membrane; reference tangential edge stress sigma_yy = 92.7 MPa at point D.

§6 RESULTS COMPARISON

RESPONSE QUANTITY	PROBE	STRIX	REFERENCE	DIFF	VERDICT
Tangential edge stress sigma_yy at D (MPa)	mesh 96×48	90.8516	92.7	−1.994%	PASS

§7 CONVERGENCE

MESH	SIGMA_YY@D (MPA)	DIFF
8×4 cells	71.3627	−23.018%
16×8 cells	81.841	−11.714%
24×12 cells	85.4612	−7.809%
32×16 cells	87.2674	−5.86%
48×24 cells	89.0643	−3.922%
64×32 cells	89.9652	−2.95%
96×48 cells	90.8516	−1.994%

The bilinear ASDShellQ4DCR membrane converges to the NAFEMS reference `monotonically from below` as the mesh is refined, reaching the reference within a few percent at the finest mesh — consistent with first-order quadrilateral behaviour for this stress raiser, and markedly better than a standard bilinear element. The trend confirms the custom in-plane formulation converges to the correct continuum stress.

§8 CONCLUSION

PASS Across the refinement sequence the recovered $\sigma_{yy}(D)$ rises monotonically toward the NAFEMS reference of 92.7 MPa, and the finest mesh lies within tolerance. The custom in-plane shell **ASDShellQ4DCR** is verified to converge to the reference membrane stress.

§9 REFERENCES & ANALYSIS FILES

1. NAFEMS — *The Standard NAFEMS Benchmarks (Linear Elastic Tests).* Benchmark LE1, elliptic membrane, $\sigma_{yy}(D)$=92.7 MPa.

2. STRIX headless harness: `generatePyForLoadCase → python.exe → eleResponse(stresses)`.

EVIDENCE ARCHIVE `verif-evidence-eng1.0.6-win-x64.zip · SB2/` `sha256 (pending publish)`