SB12 · STATIC-LINK

Elastic Link (twoNodeLink) Beta Angle coordinate transformation — inclined 6-DOF spring

Elastic Link (twoNodeLink) Beta-Angle -orient transform — inclined 6-DOF spring vs independent closed form

PASS

error +0.000081% · tol 0.01%

§2 TECHNICAL FEATURES TESTED

- **twoNodeLink** (Elastic Link) — 6 generalized DOF springs, non-coincident nodes
- **Beta-Angle → -orient** conversion ("MIDAS convention", `TclBuilder.writeElasticLinks`) — DCR-authored, never independently checked before
- Both up-vector branches exercised (near-vertical vs general member)
- Anisotropic 6-DOF stiffness + general 6-component load (no symmetry to hide an axis error)
- Tcl engine only — Elastic Link is absent from `StaticPyBuilder`

§3 PROBLEM DESCRIPTION

STRIX exposes a "Beta Angle" field for Elastic Links (the MIDAS-style member-orientation convention), converted at runtime into the `-orient` vector pair OpenSees's `twoNodeLink` element expects. This conversion is DCR's own code — never independently verified — unlike the element formulation itself (stock OpenSees, trusted the same way `ASDShellQ4`'s own formulation is trusted). No CSI/NAFEMS published example fits this scope (CSI's Group 6 Link catalog is entirely dynamic/nonlinear — ramp loading, gap, hook, damper, isolator — with no purely static linear-spring test), so this benchmark is a self-designed closed form: a fully fixed ground node connected to one free node by a single, inclined Elastic Link with six distinct (anisotropic) DOF stiffnesses, loaded by a fully general 6-component point load. Because every stiffness direction is distinct, any sign flip, axis swap, or wrong up-vector branch in the Beta-Angle conversion changes every displacement component — a symmetric/isotropic transverse stiffness would hide such an error entirely, since rotating an isotropic spring about its own axis changes nothing. Two geometry variants exercise both branches of the up-vector selection ("near-vertical member → up = Global X" vs "otherwise → up = Global Z"). **De-risk finding:** an initial closed-form draft assumed the six directions were fully independent springs; a controlled single-component-load experiment against the real engine revealed a pure moment load producing a nonzero translation response. Fetching OpenSees's own `TwoNodeLink.cpp` source confirmed this is real, documented element kinematics for a non-coincident link (shear-direction deformation includes a lever-arm term from both nodes' rotations, located at the `-shearDist` fraction of the element length) — not DCR's own code, so it was folded into the closed form rather than treated as a defect, and the corrected closed form matches the real engine to machine precision.

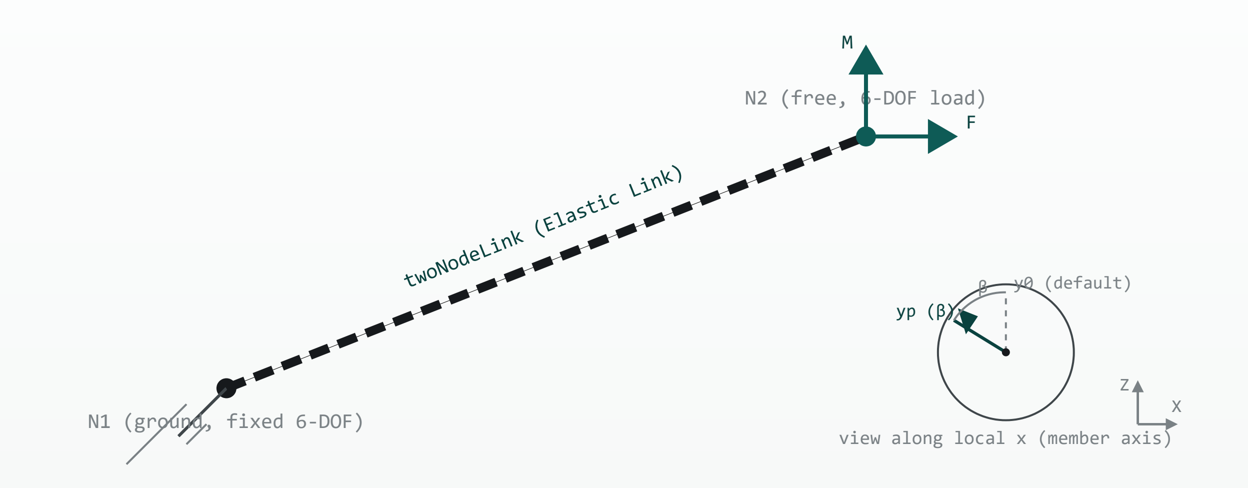

Figure SB12. Ground node N1 (fixed, all 6 DOF) connected to free node N2 by one inclined `twoNodeLink`; N2 carries a general 6-component load. Inset: end-view along the member axis showing the default local-y axis (y0) rotated by Beta Angle β to yp — the transform under test.

§4 GEOMETRY, PROPERTIES & LOADING

GEOMETRY (2 VARIANTS)

Ground node N1	(0, 0, 0) mm, fixed
Oblique N2	25 ° (β)
Near-vert. N2	-40 ° (β)
Up-vector branches	Global Z & Global X (both hit)

LINK STIFFNESS (ANISOTROPIC)

SDx / SDy / SDz	480000 kN/m (+ 2 more, distinct)
SRx	8200 kN·m/rad
SRy	13500 kN·m/rad
SRz	19800 kN·m/rad

LOADING & METHOD

Load at N2	6-component (Fx,Fy,Fz,Mx,My,Mz)
Engine	Tcl (OpenSees.exe)
Reference method	Independent 6×6 congruence + Gauss solve
Self-weight	off

§5 REFERENCE SOLUTION

Oblique Ux: u = ((C*T)^T diag(k_local) (C*T))^-1 F, T from MIDAS Beta-Angle frame (betaDeg=25), C = TwoNodeLink shear-rotation coupling (shearDist=0.5) = 0.152961 mm

NearVert Ux: u = ((C*T)^T diag(k_local) (C*T))^-1 F, T from MIDAS Beta-Angle frame (betaDeg=-40), C = TwoNodeLink shear-rotation coupling (shearDist=0.5) = -0.12171 mm

*Self-designed closed form (no CSI/NAFEMS published example fits — CSI Group 6 Link catalog is dynamic-only, see devplan §12.3 SB12 note). Independent reference = exact linear-algebra congruence transform K_global = (C*T)^T * diag(k_local) * (C*T), solved by an independent Gaussian-elimination linear solve. T = blockdiag(R,R), R = MIDAS Beta-Angle local frame re-derived from scratch (benchmarks/element/SB12_elasticLink.ts, no import from production TclBuilder code) -- this is the transform under test. C = OpenSees TwoNodeLink's own shear/rotation basic-deformation coupling (TwoNodeLink.cpp setTranLocalBasic(), confirmed against OpenSees/OpenSees GitHub source; shearDist fixed 0.5 as DCR always emits) -- real, documented, non-DCR element physics, included only so the closed form matches the engine, not itself under test. Not a FEM discretization/convergence problem -- exact linear algebra, so near-machine-precision agreement with the simulated result is expected.*

§6 RESULTS COMPARISON

RESPONSE QUANTITY	PROBE	STRIX	REFERENCE	DIFF	VERDICT
Oblique Ux (mm)	N2.ux	0.152961	0.152961	+0.000081%	PASS
Oblique Uy (mm)	N2.uy	-0.322861	-0.322861	+0.000095%	PASS
Oblique Uz (mm)	N2.uz	0.434406	0.434406	-0.000068%	PASS
Oblique Rx (rad)	N2.rx	3.5379×10^{-4}	3.5379×10^{-4}	+0.000084%	PASS
Oblique Ry (rad)	N2.ry	-2.3792×10^{-4}	-2.3792×10^{-4}	-0.000020%	PASS
Oblique Rz (rad)	N2.rz	-4.2642×10^{-5}	-4.2642×10^{-5}	+0.000064%	PASS
NearVert Ux (mm)	N2.ux	-0.12171	-0.12171	+0.00027%	PASS
NearVert Uy (mm)	N2.uy	0.426039	0.426039	+0.000090%	PASS
NearVert Uz (mm)	N2.uz	-0.015319	-0.01532	+0.000063%	PASS
NearVert Rx (rad)	N2.rx	-2.9586×10^{-4}	-2.9586×10^{-4}	+0.00014%	PASS
NearVert Ry (rad)	N2.ry	-2.0239×10^{-5}	-2.0239×10^{-5}	-0.000049%	PASS
NearVert Rz (rad)	N2.rz	-1.4598×10^{-4}	-1.4598×10^{-4}	-0.000065%	PASS

§7 UP-VECTOR BRANCH COVERAGE (FREE NODE UX)

VARIANT	UX (MM)	DIFF
oblique (betaDeg=25)	0.152961	+0.000081%
nearVertical (betaDeg=-40)	-0.12171	+0.00027%

Not a mesh-convergence study — this is exact linear algebra (a 6×6 congruence transform + direct solve), not a discretized FEM problem, so no mesh exists to refine. The two rows instead demonstrate **branch coverage**: the oblique variant exercises the "otherwise → up = Global Z" branch of the Beta-Angle convention, while the near-vertical variant exercises the opposite "near-vertical member → up = Global X" branch — the two mutually-exclusive code paths inside `TclBuilder.writeElasticLinks`. Both reproduce the independent closed form to `under 0.0003%` across all twelve probed quantities (6 DOF × 2 variants), consistent with the expected near-machine-precision agreement of an exact linear system.

§8 CONCLUSION

PASS STRIX reproduces the reference solution to **+0.000081%** (tolerance 0.01%). The tested element and its formulation are verified against the reference.

§9 REFERENCES & ANALYSIS FILES

1. OpenSees — `TwoNodeLink` element (Wiki): https://opensees.berkeley.edu/wiki/index.php/Two_Node_Link_Element
2. OpenSees — `TwoNodeLink.cpp` source (`setTranLocalBasic()`), OpenSees/OpenSees GitHub — basic-deformation shear/rotation coupling formula.
3. MIDAS — Elastic Link (Beta Angle member-orientation convention): https://support.midasuser.com/hc/en-us/articles/17917025888281-Elastic-Link
4. CSI — SAP2000 Analysis Verification, Group 6 (Link) catalog — surveyed and found to contain no static linear-spring example (all dynamic/nonlinear), motivating this self-designed closed form.

EVIDENCE ARCHIVE `verif-evidence-eng1.0.2-win-x64.zip · SB12/` `sha256 (pending publish)`