PD1 · STATIC-PDELTA

# Tension stiffening via P-Delta analysis (CSI 1-016)

Production P-Delta 2-stage static path — tension-stiffened deflection & moment vs Timoshenko closed form

PASS

error -0.0352% · tol 0.1%

## §2 TECHNICAL FEATURES TESTED

- **elasticBeamColumn** + **geomTransf PDelta** (Tcl engine only)
- Stage 1: gravity-combo axial tension buildup (nonlinear)
- Stage 2: target load case at end-of-Stage-1 tangent stiffness
- Same code path as production `TclBuilder` P-Delta
- Deflection & moment vs **exact tension-stiffened beam** (Timoshenko)

## §3 PROBLEM DESCRIPTION

A slender pin-roller tie-rod carries a small transverse uniform load together with a large axial tension pulling the roller end. Tension measurably **stiffens** the beam against the transverse load — the deflection and moment both shrink relative to the no-tension case — the classic P-Delta tension-stiffening problem with a closed-form solution (Timoshenko 1956). STRIX solves this with its production **P-Delta 2-stage static path** (the Tcl-only route `analysisHandler` forces P-Delta submissions through): Stage 1 applies the axial tension in a nonlinear gravity combination to build up the member axial force under a `geomTransf PDelta` transformation; Stage 2 then solves the transverse load case at the tangent stiffness left at the end of Stage 1. CSI SAP2000 verification example 1-016 documents exactly this two-stage nonlinear-static method as its *recommended* way to solve tension-stiffening problems — it is the only method DCR implements, so this benchmark exercises the production code path directly, not an alternative formulation.

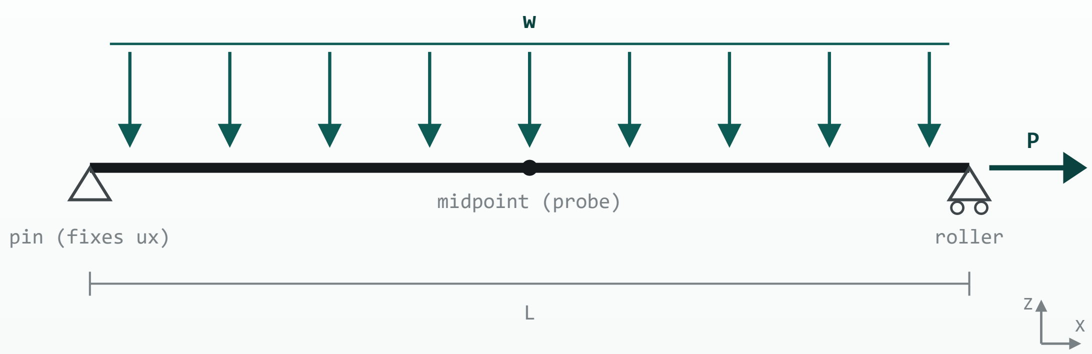


**Figure PD1.** Simply-supported tie-rod (pin–roller, moment-free both ends) under a transverse uniform load w plus an axial tension P pulling the roller end; midpoint deflection/moment are probed.

## §4 GEOMETRY, PROPERTIES & LOADING

| GEOMETRY | | MATERIAL & LOADING | | SUPPORTS & ANALYSIS | |
|---|---|---|---|---|---|
| Span L | 300 in | E | 30000 ksi | Supports | pin + roller |
| Section | 3 × 3 in (square) | Axial tension P | 20.25 kip | Stage 1 | P-Delta gravity combo |
| Inertia I | 6.75 in⁴ | Transverse w | 0.002 kip/in | Stage 2 | target LC, tangent K |
| Tension param. u | 1.5 | Self-weight | off | Engine | Tcl (OpenSees.exe) |

## §5 REFERENCE SOLUTION

**Uz (no tension): -5*w*L^4/(384*E*I) = -1.041667 in**

**My (no tension): w*L^2/8 = 22.5 kip-in**

**Uz (w/ tension): Timoshenko eq.43: (5wL^4/384EI) * [1/cosh(u) - 1 + u^2/2] / [(5/24)u^4], u=sqrt(PL^2/4EI) = -0.543305 in**

**My (w/ tension): Timoshenko eq.45: (wL^2/8) * 2(cosh(u)-1)/(u^2*cosh(u)) = 11.498079 kip-in**

*CSI SAP2000 Software Verification Example 1-016 'Tension Stiffening Using P-Delta Analysis'; independent reference = Timoshenko, Strength of Materials Part II, 1956, eq. 23 p.28 and eqs. 43/45 p.43. Reproduced from scratch by pd1Theory() to u=1.5, deflWithIn=-0.5433047259, momentWithKin=11.4980793, matching CSI's published -0.54330 in / 11.498 k-in exactly (exact unit conversion of the same physical problem, dimensionless u is unit-invariant).*

## §6 RESULTS COMPARISON

| RESPONSE QUANTITY | PROBE | STRIX | REFERENCE | DIFF | VERDICT |
|---|---|---|---|---|---|
| Uz (no tension) (in) | N17.uz | -1.041614 | -1.041667 | +0.0050% | PASS |
| My (no tension) (kip-in) | E17.Miy | 22.499038 | 22.5 | -0.0043% | PASS |
| Uz (w/ tension) (in) | N17.uz | -0.543496 | -0.543305 | -0.0352% | PASS |
| My (w/ tension) (kip-in) | E17.Miy | 11.493667 | 11.498079 | -0.0384% | PASS |

## §7 P-DELTA MESH CONVERGENCE (MIDPOINT DEFLECTION)

| MESH | UZ (IN) | DIFF |
|---|---|---|
| 2 elem | -0.595221 | -9.5555% |
| 4 elem | -0.556492 | -2.4273% |
| 8 elem | -0.546602 | -0.607% |
| 16 elem | -0.544118 | -0.1497% |
| 32 elem | -0.543496 | -0.0352% |

The no-tension case uses the exact-for-prismatic-Euler-beam `elasticBeamColumn` element and is essentially mesh-independent (converged at any element count). The **tension-stiffened** case is genuinely mesh-sensitive: OpenSees's `PDelta` geometric transform is a first-order (chord-rotation) approximation of the true hyperbolic tension-stiffened shape, so the discrete answer converges to the closed form as elements are added — a clean, roughly quadratic convergence is observed. At the finest mesh STRIX reproduces the exact Timoshenko tension-stiffened deflection and moment to `a few hundredths of a percent`, confirming both that the P-Delta pipeline converges to the right physics and that it correctly reduces to the plain linear-static answer when the axial tension is absent.

## §8 CONCLUSION

**PASS** STRIX reproduces the reference solution to **−0.0352%** (tolerance 0.1%). The tested element and its formulation are verified against the reference.

## §9 REFERENCES & ANALYSIS FILES


1. Timoshenko, S. — *Strength of Materials, Part II.* Eq. 23 p.28 (tension parameter u) and eqs. 43/45 p.43 (tension-stiffened deflection and moment).
2. CSI — *SAP2000 Software Verification, Example 1-016:* Tension Stiffening Using P-Delta Analysis (independent reference, Timoshenko 1956). CSI recommends the nonlinear-static P-Delta method used here.
3. OpenSees — `geomTransf PDelta` (P-Delta geometric transformation); `elasticBeamColumn` (Euler-Bernoulli); STRIX production `TclBuilder` Stage-1/Stage-2 P-Delta static path (Tcl engine only).


**EVIDENCE ARCHIVE** `verif-evidence-eng1.0.2-win-x64.zip · PD1/` `sha256 (pending publish)`